INFORMATION COMPRESSION IN BIOLOGICAL RHYTHM BY WAVELET ANALYSIS

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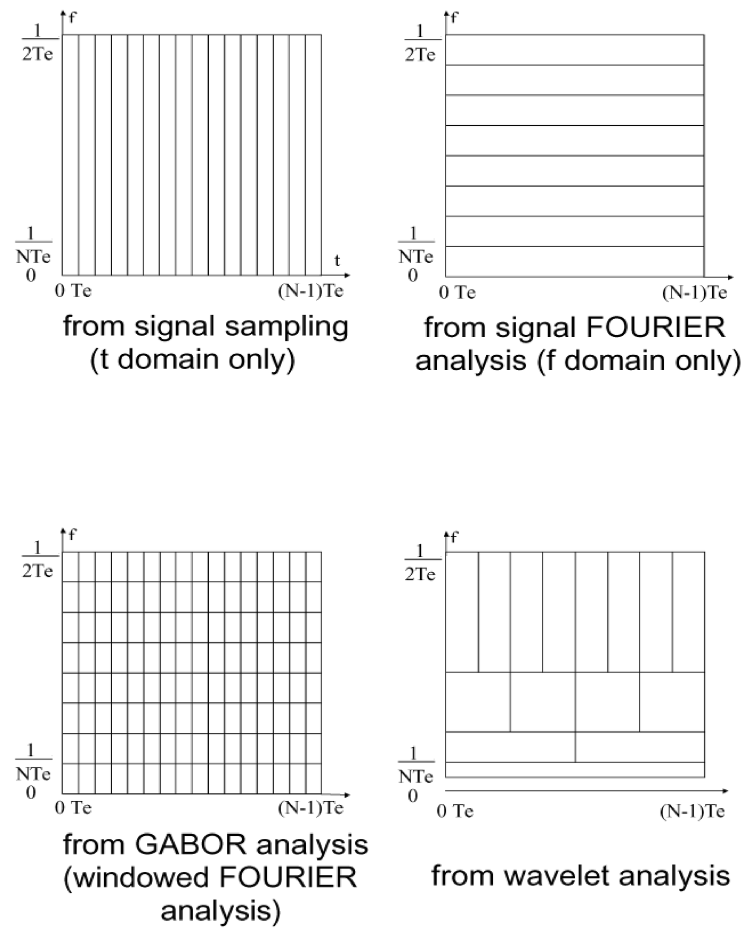
WAVELET TRANSFORM

Time/frequency domains

Representation of rhythms in time/frequency (t/f) domain is linked to data processing.



Only the last two cases allow simultaneous time and frequency study; but wavelet transform optimises t/f domain.

Continuous case

From the wavelet analysing Ψ defined with the conditions :

$$C = \int_{-\infty}^{+\infty} \frac{|\Psi(n)|^2}{|n|} dn$$

the wavelet transform is obtained :

$$W_f(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} f(t)\Psi\left(\frac{t-b}{a}\right)dt$$

$$f(t) = \frac{1}{C}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \Psi(a,b) W_f(a,b)\frac{da}{a^2}db$$

where a is a dilatation coefficient (f domain)
b a time translation coefficient (t domain)

Discrete case

Rather than integral calculus, discrete case uses matrix

$$\prod_i P_i T_i$$

where Pi is a permutation matrix and Ti a transform matrix defined with known DAUBECHIES coefficients.

This WT algorithm is extremely fast like FFT but must also use 2^p samples.

Compression method operator:

$$WT^{-1} \circ Th \circ WT$$

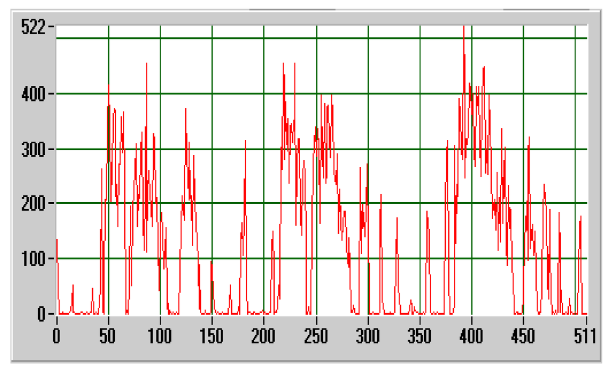
On the reconstruction the Th operator is defined by a specifically threshold criterion.

WAVELET ANALYSIS APPLIED TO BIOLOGICAL RHYTHMS

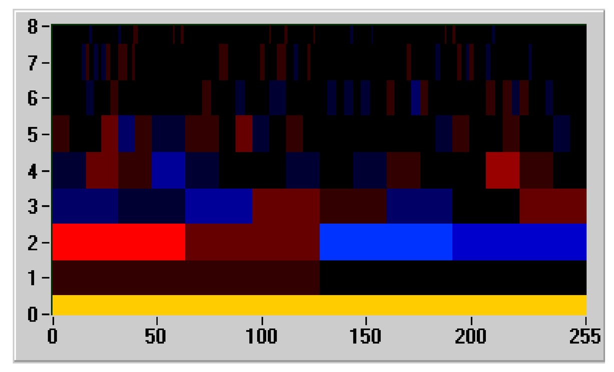
Biological rhythm analysis and reconstruction

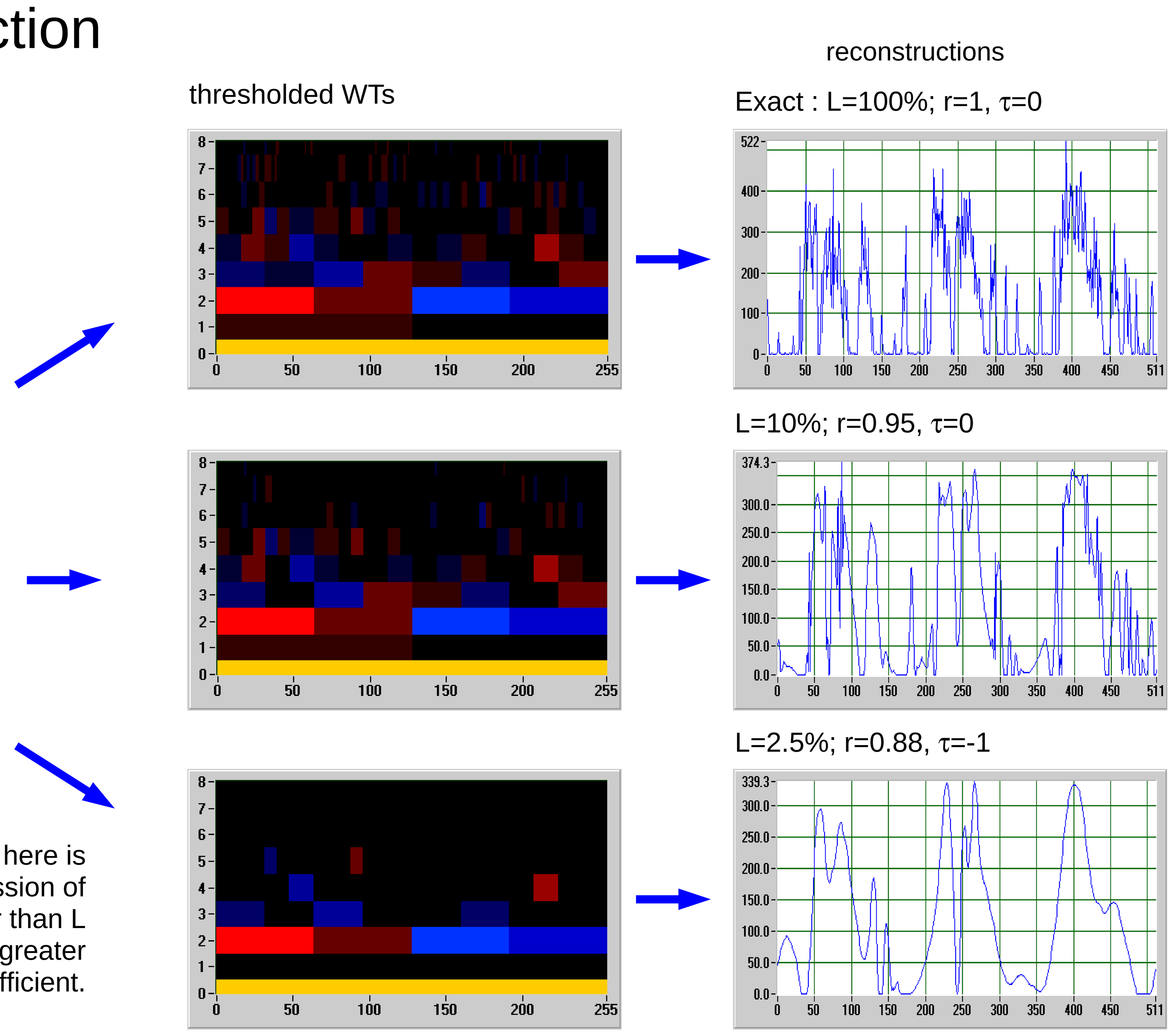
Examples from rhythm activity of inbred strains of mice (in L:D 12:12) defined with 512 samples in a 3 day window.

temporal original signal



wavelet transform

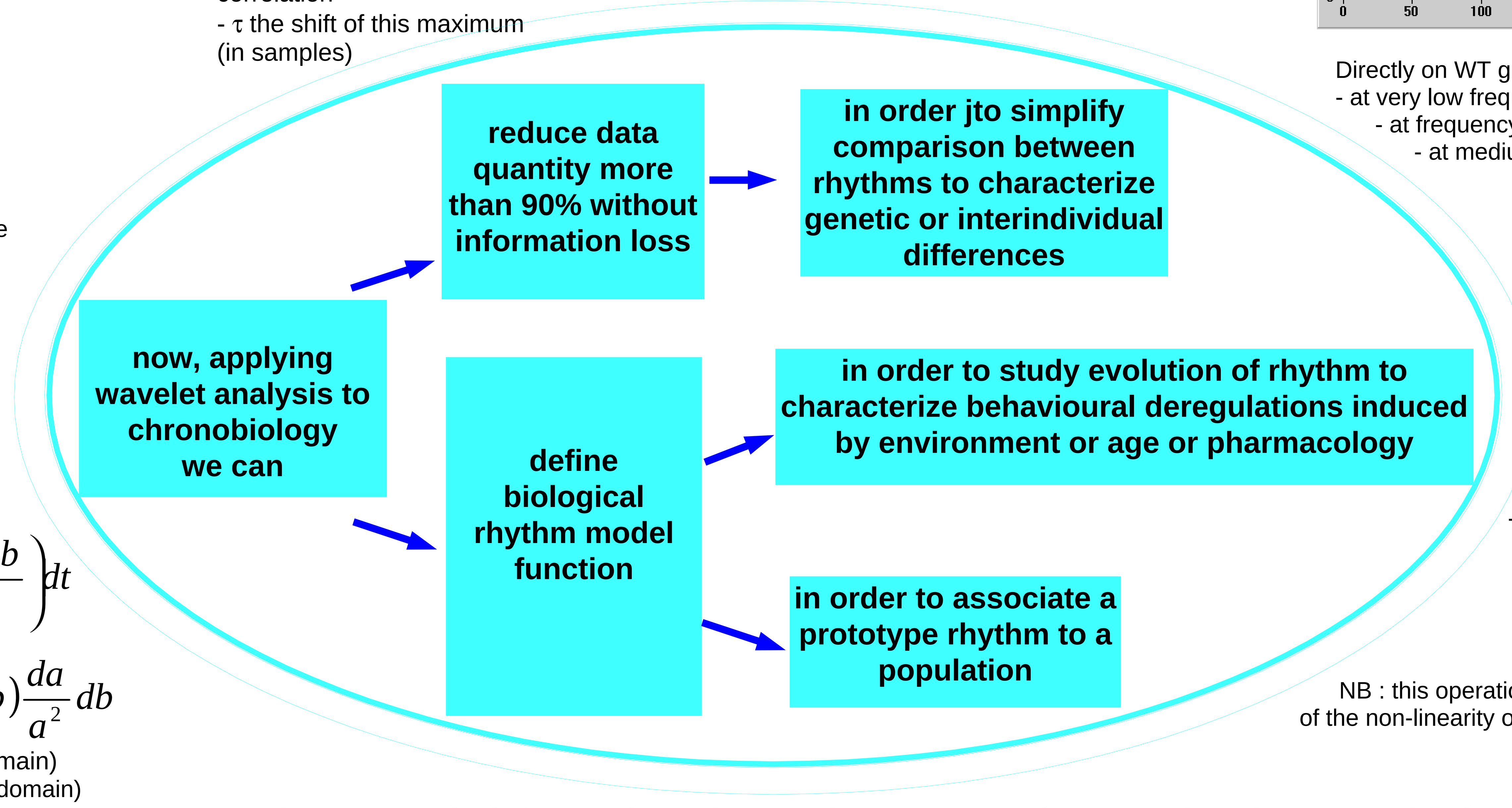




Reconstruction efficiency is measured by correlation between original and reconstructed signals.
- r is the maximum normalised correlation
- τ the shift of this maximum (in samples)

Threshold criterion here is obtained by suppression of coefficients smaller than L in % of the greater coefficient.

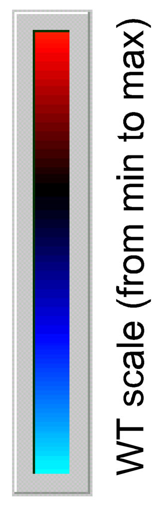
Directly on WT graph we can observe :
- at very low frequency (at the bottom) the stationarity
- at frequency corresponding to 1 day, the intensity of circadian component
- at medium frequency a pulsatile activity phased with light/dark synchronizer



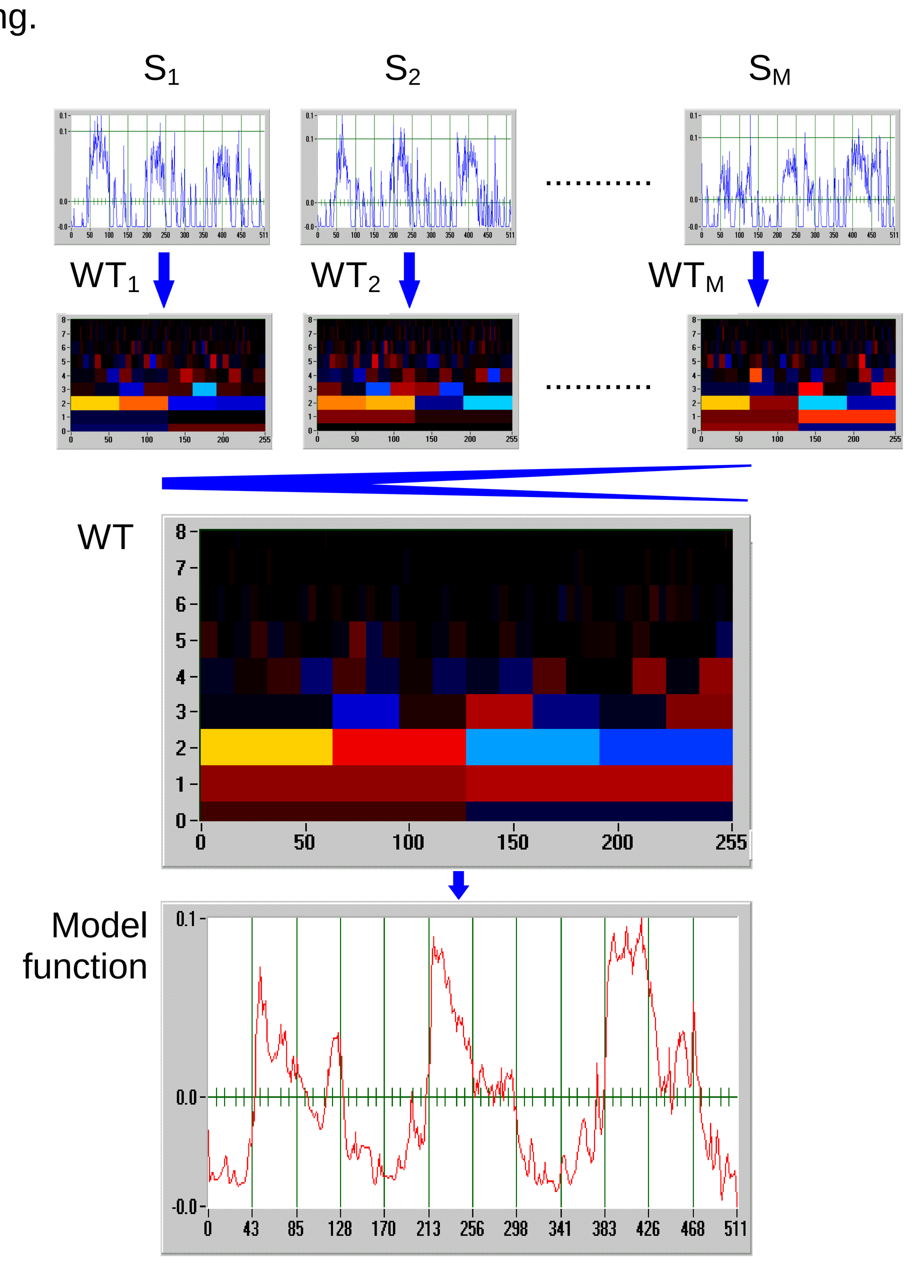
Biological rhythm model

Method :
- in a population each subject rhythm S_i is analysed
- on each WT_i an identical Th function is applied
- the WT model function is the mean of these WT_i
- Inverse wavelet transform leads to the model function.

$$WT^{-1} \circ \left[\frac{1}{M}\sum_{i=0}^{M-1} Th \circ WT \circ R_i\right]$$



NB : this operation is fundamentally different from a simple averaging by introduction of the non-linearity of thresholding.



Using model function in order to evaluate rhythm variations

Model function validation

Correlation (r and τ) between each subject and model function evaluate the quality of the model function.
Calculus are performed on a 16 subject population.

	Correlation r	Shift τ
Mean	0.66	0.44
Standard devia-	0.08	1.75

F1 comparisons
- B6 : C57/Bl6 strain
- C : Balb/C strain
- CB and BC : reciprocal F1

F1 comparisons are measured by the correlation coefficient r.
Here, reciprocal F1 are identical what verify non maternal effect.

r	B6	C	CB	BC
B6	1	0.70	0.85	0.86
C		1	0.72	0.77
CB			1	0.91
BC				1

Experimental and environmental variations
- N : Normal condition (reference)
- R : Retest condition (1 week later)
- D : food-Deprivation condition (3 days)
- I : Isolated condition (2 weeks)

Those results show light variations *vs* experimental conditions

r	N	R	D	I
B6	1	0.77	0.75	0.77
C	1	0.73	0.71	0.80